General Algorithm Primitives

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Topics

- **Sorting**
  - Sorting networks
- **Search**
  - Binary search
  - Nearest neighbor search
- **Stream Filtering**

Assumptions

- **Data organized into 1D arrays**
- **Rendering pass == screen aligned quad**
  - Not using vertex shaders
- **PS 2.0 GPU**
  - No data dependent branching at fragment level

Sorting

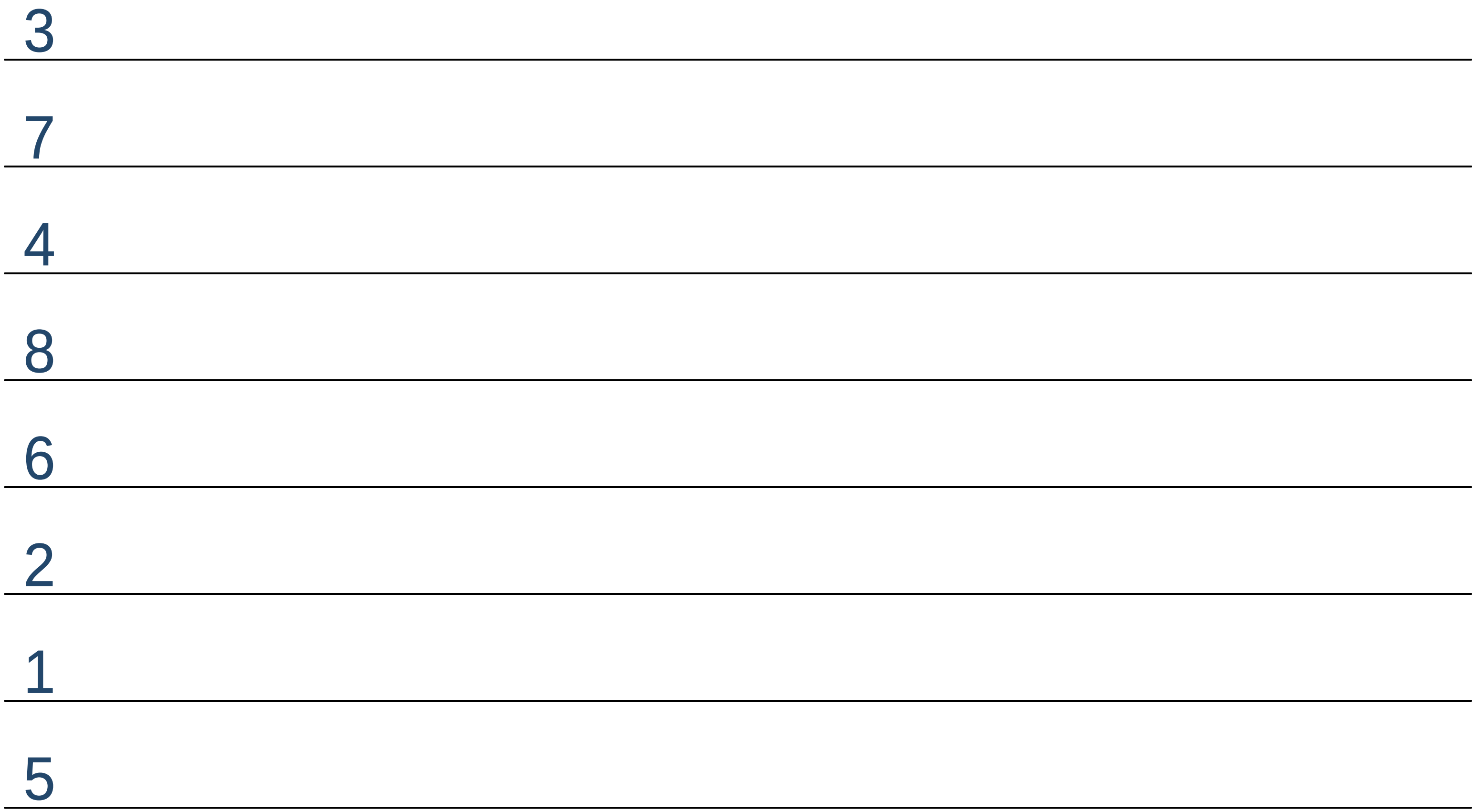
Sorting

- **Given an unordered list of elements, produce list ordered by key value**
  - Kernel: compare and swap
- **GPUs constrained programming environment limits viable algorithms**
  - Bitonic merge sort [Batcher 68]
  - Periodic balanced sorting networks [Dowd 89]

Bitonic Merge Sort Overview

- **Repeatedly build bitonic lists and then sort them**
  - Bitonic list is two monotonic lists concatenated together, one increasing and one decreasing.
    - List A: (3, 4, 7, 8) monotonically increasing
    - List B: (6, 5, 2, 1) monotonically decreasing
    - List AB: (3, 4, 7, 8, 6, 5, 2, 1) bitonic

Bitonic Merge Sort



3

7

4

8

6

2

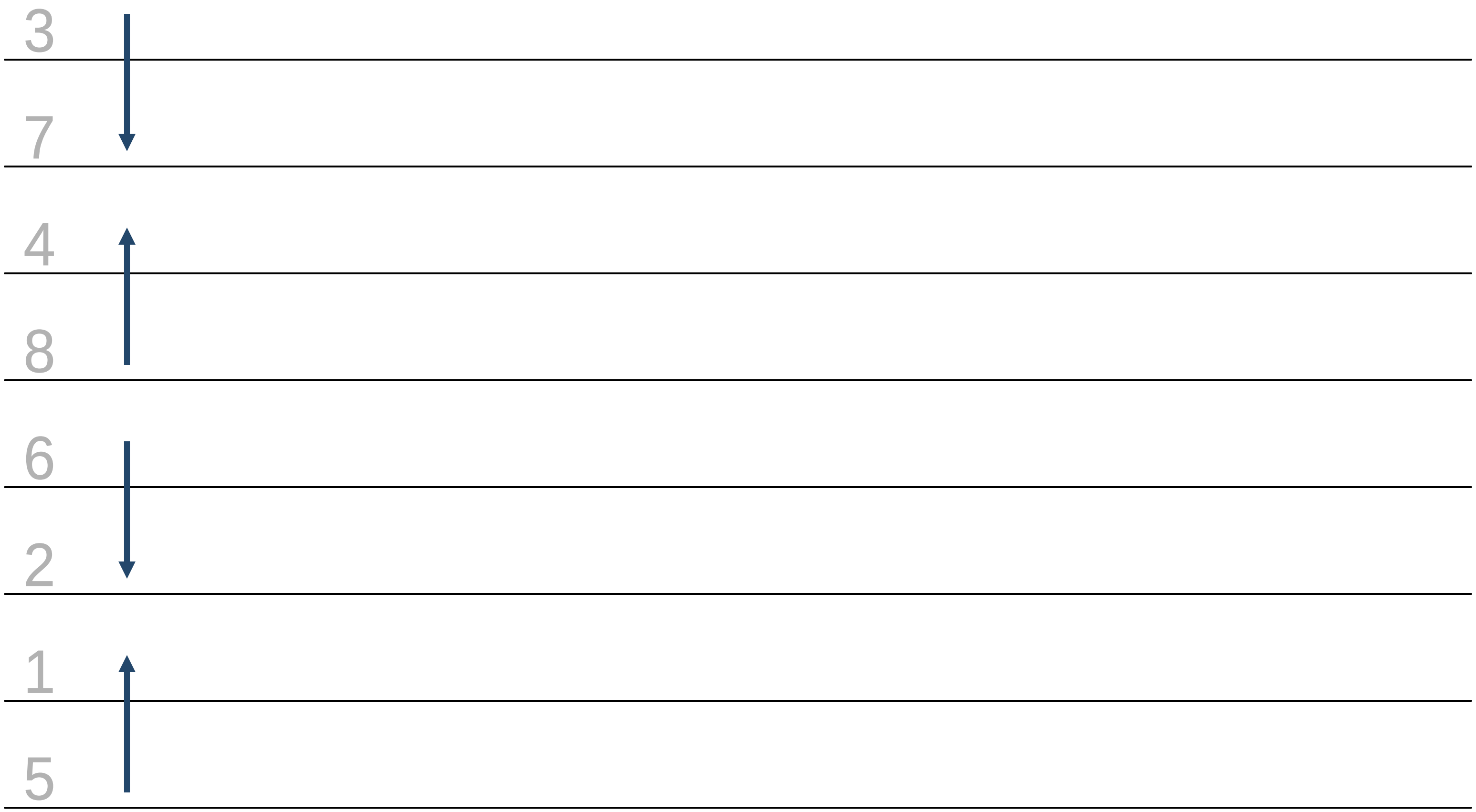
1

5

8x monotonic lists: (3) (7) (4) (8) (6) (2) (1) (5)

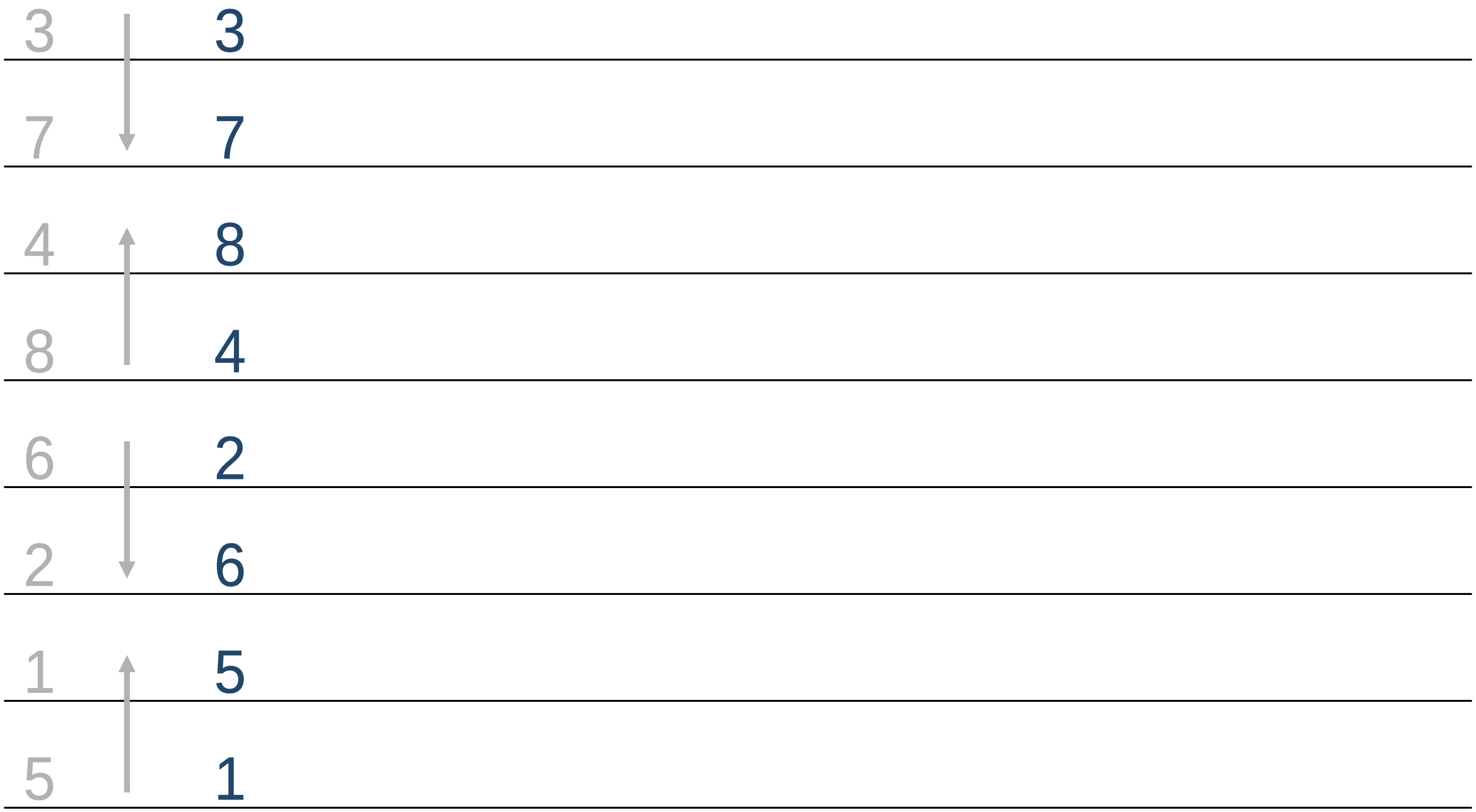
4x bitonic lists: (3,7) (4,8) (6,2) (1,5)

Bitonic Merge Sort



Sort the bitonic lists

Bitonic Merge Sort

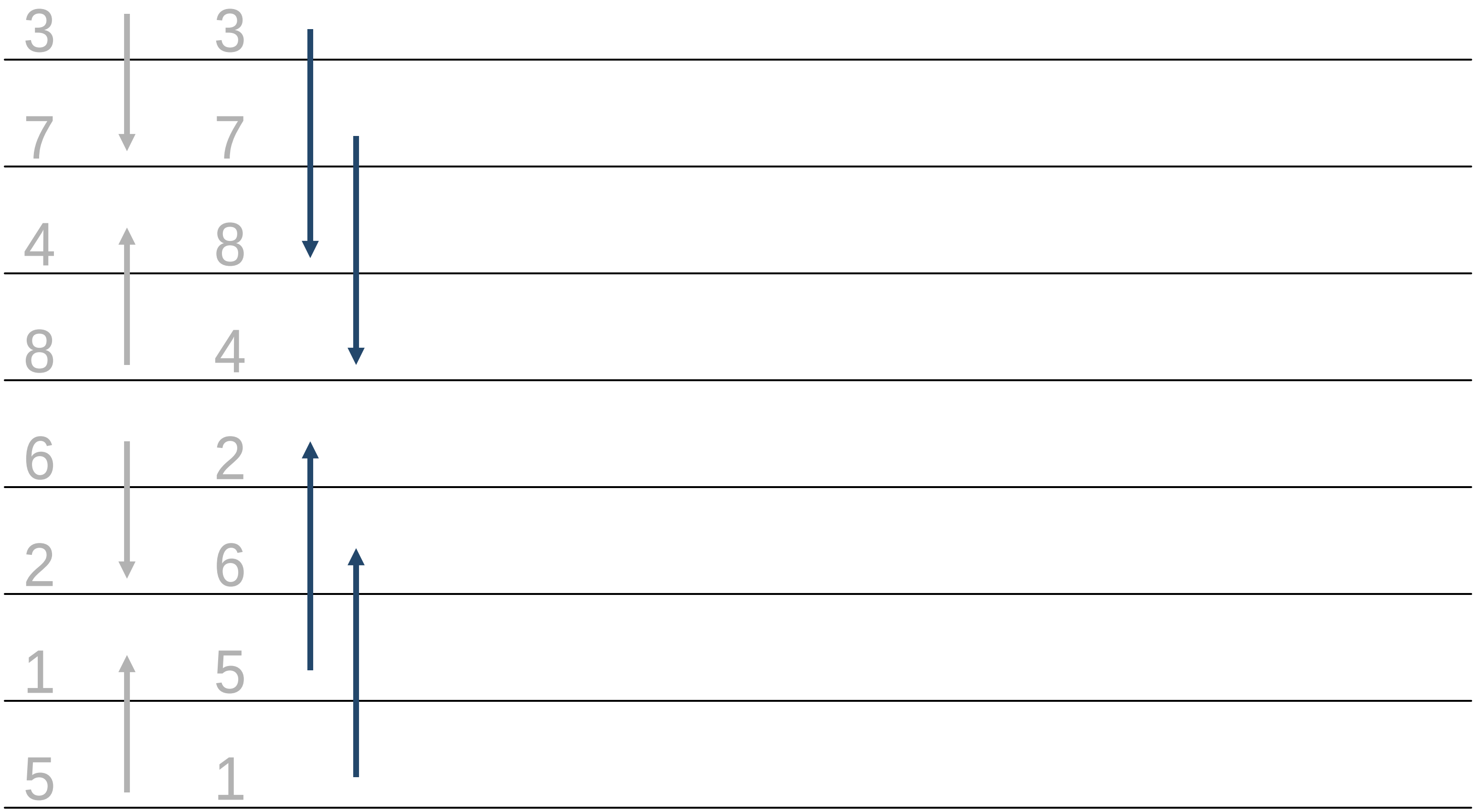


3	3
7	7
4	8
8	4
6	2
2	6
1	5
5	1

4x monotonic lists: (3,7) (8,4) (2,6) (5,1)

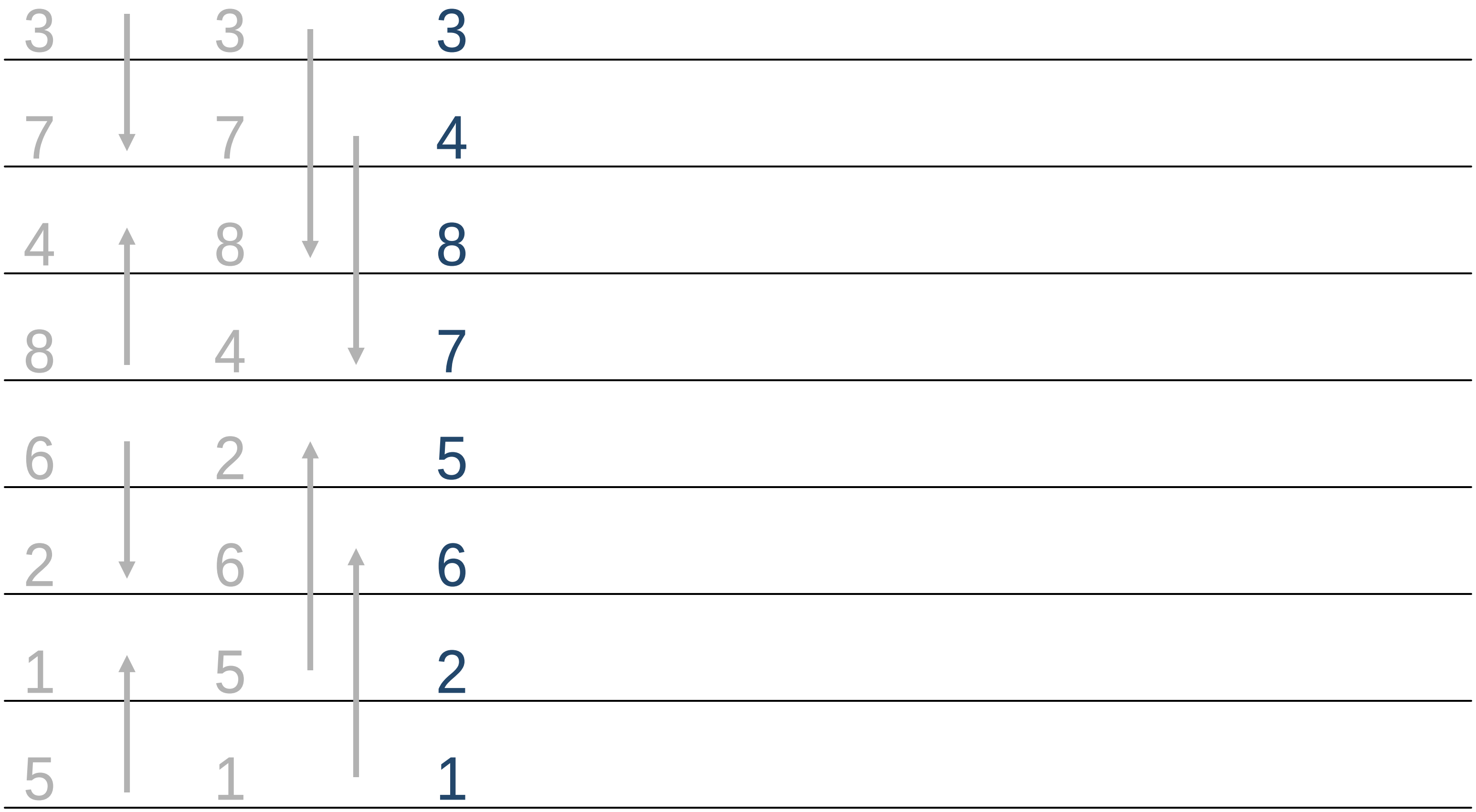
2x bitonic lists: (3,7,8,4) (2,6,5,1)

Bitonic Merge Sort



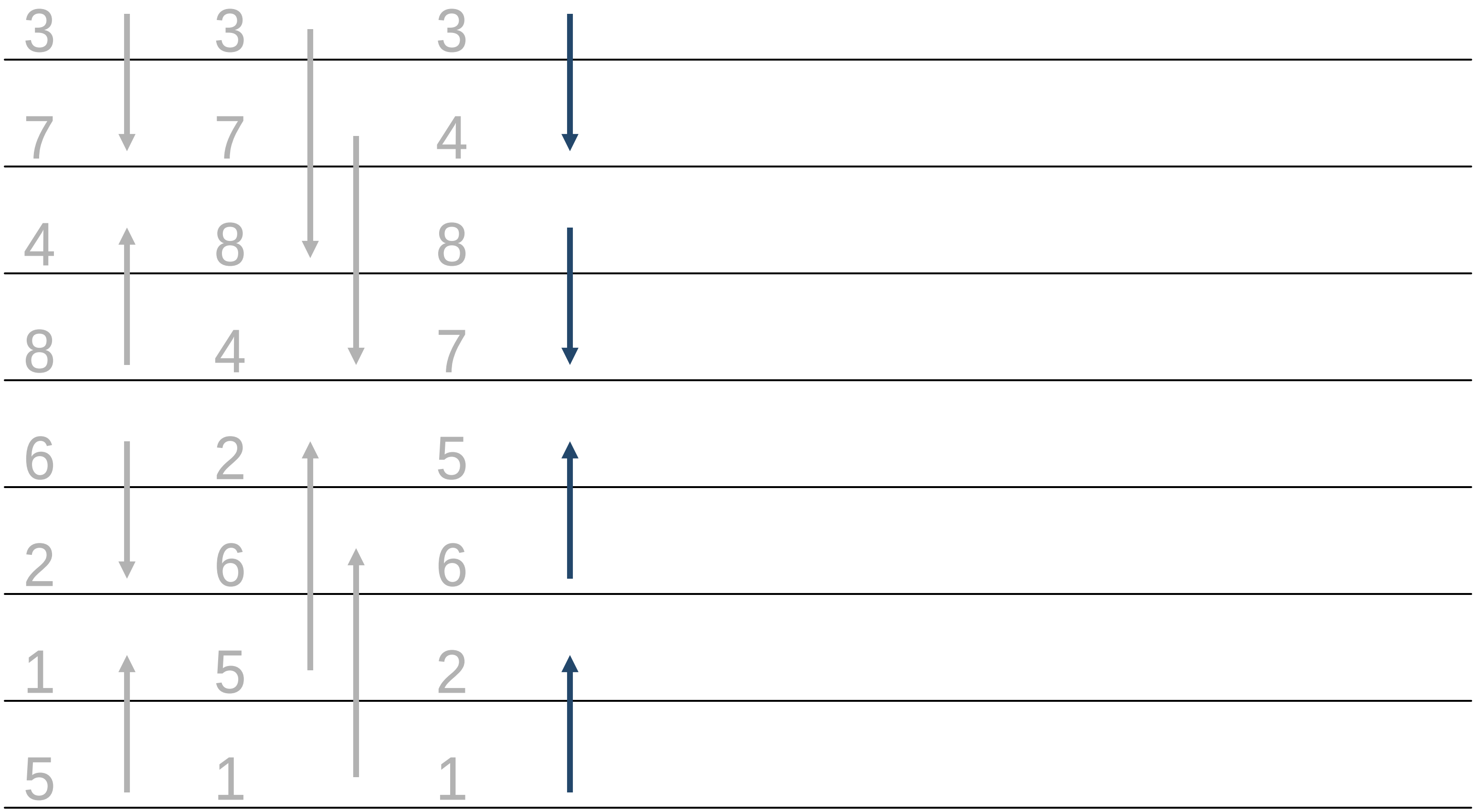
Sort the bitonic lists

Bitonic Merge Sort



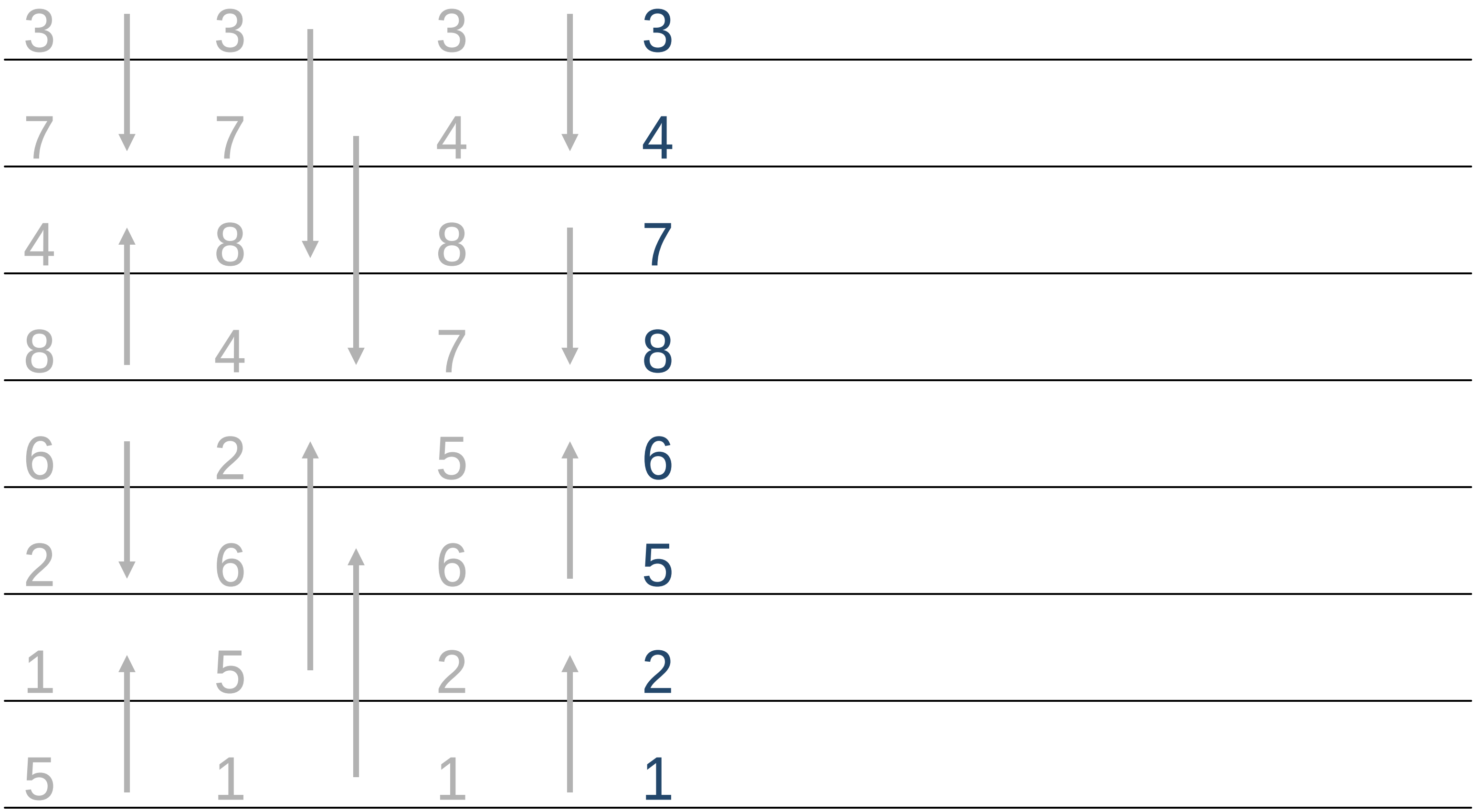
Sort the bitonic lists

Bitonic Merge Sort



Sort the bitonic lists

Bitonic Merge Sort

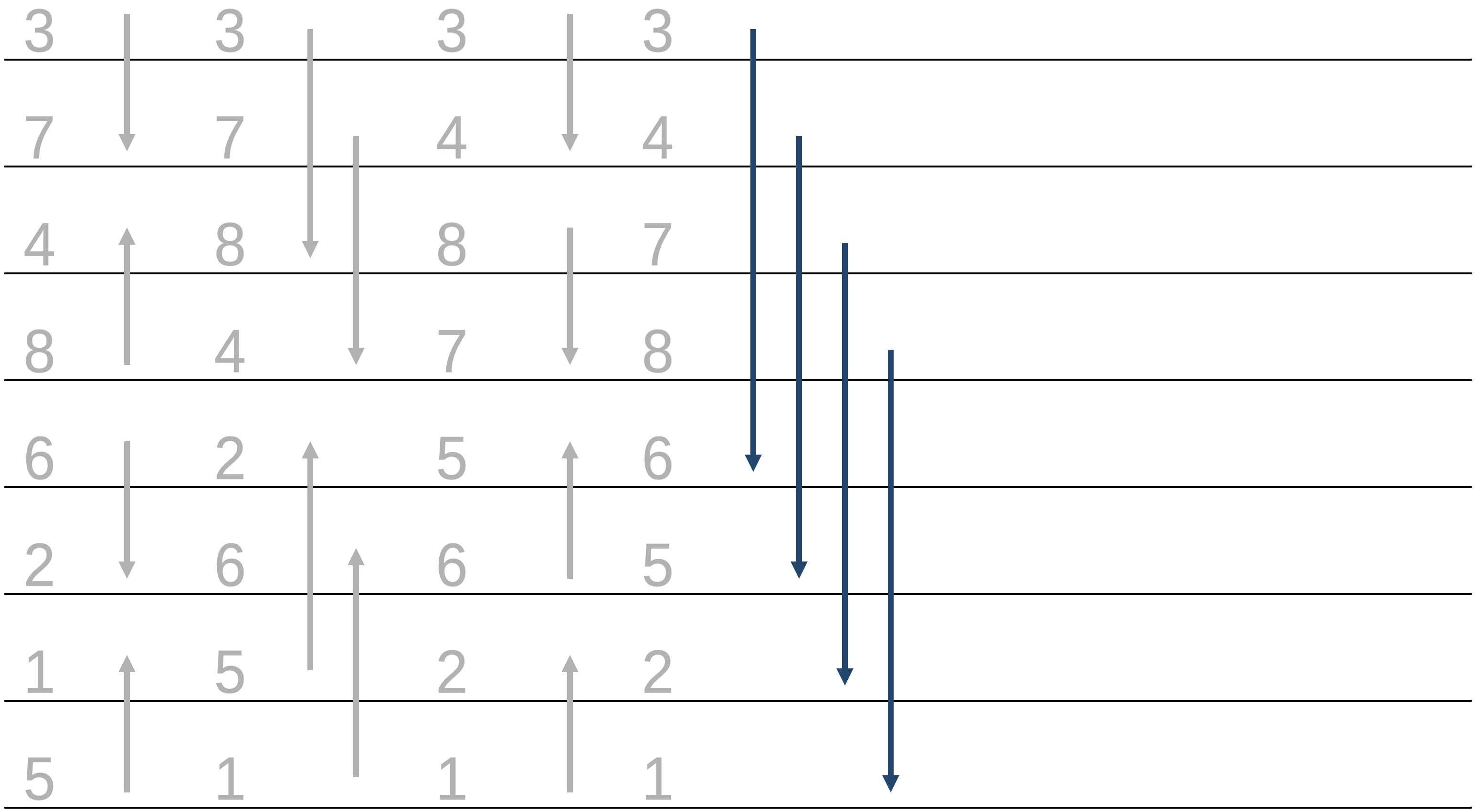


3	3	3	3
7	7	4	4
4	8	8	7
8	4	7	8
6	2	5	6
2	6	6	5
1	5	2	2
5	1	1	1

2x monotonic lists: (3,4,7,8) (6,5,2,1)

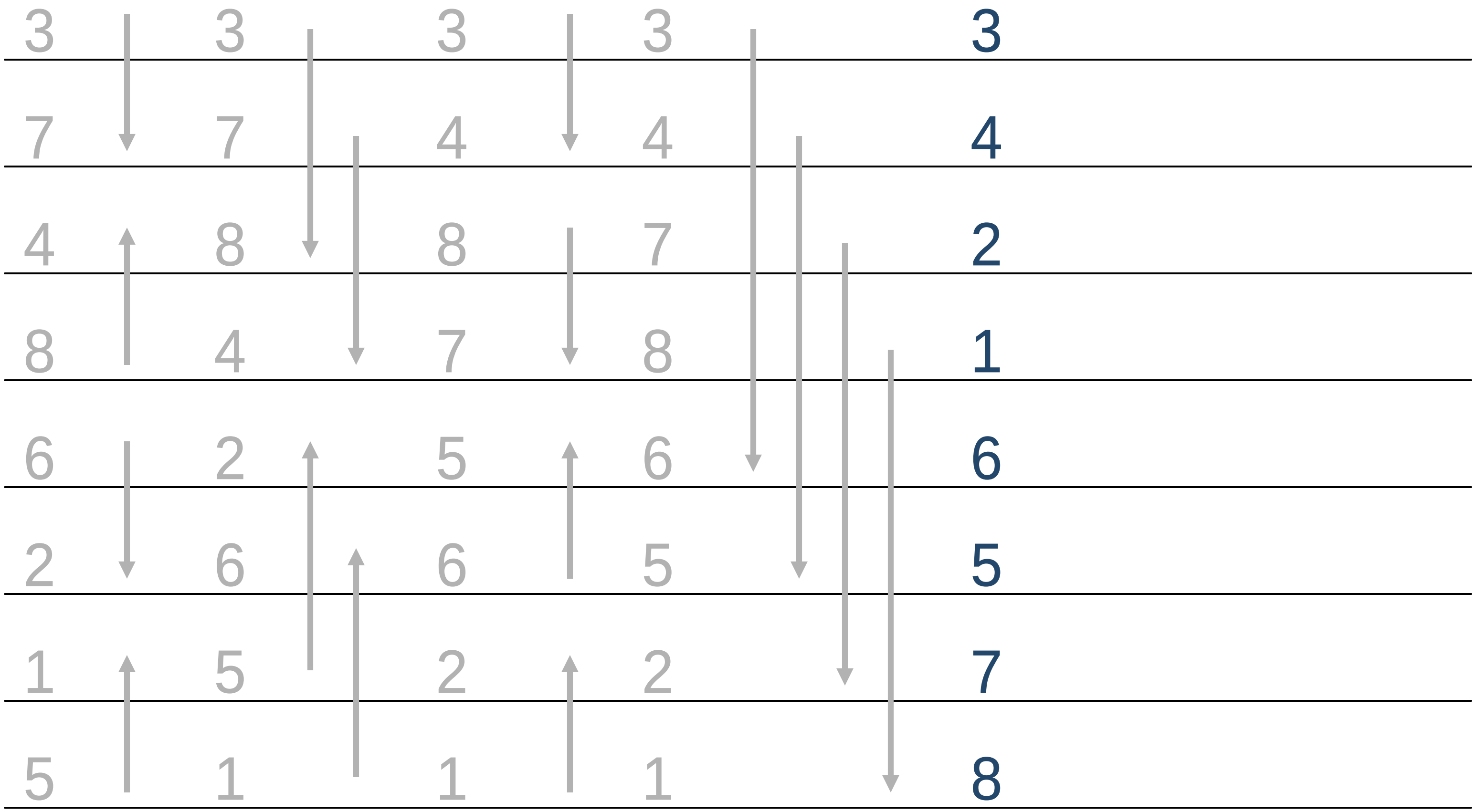
1x bitonic list: (3,4,7,8, 6,5,2,1)

Bitonic Merge Sort



Sort the bitonic list

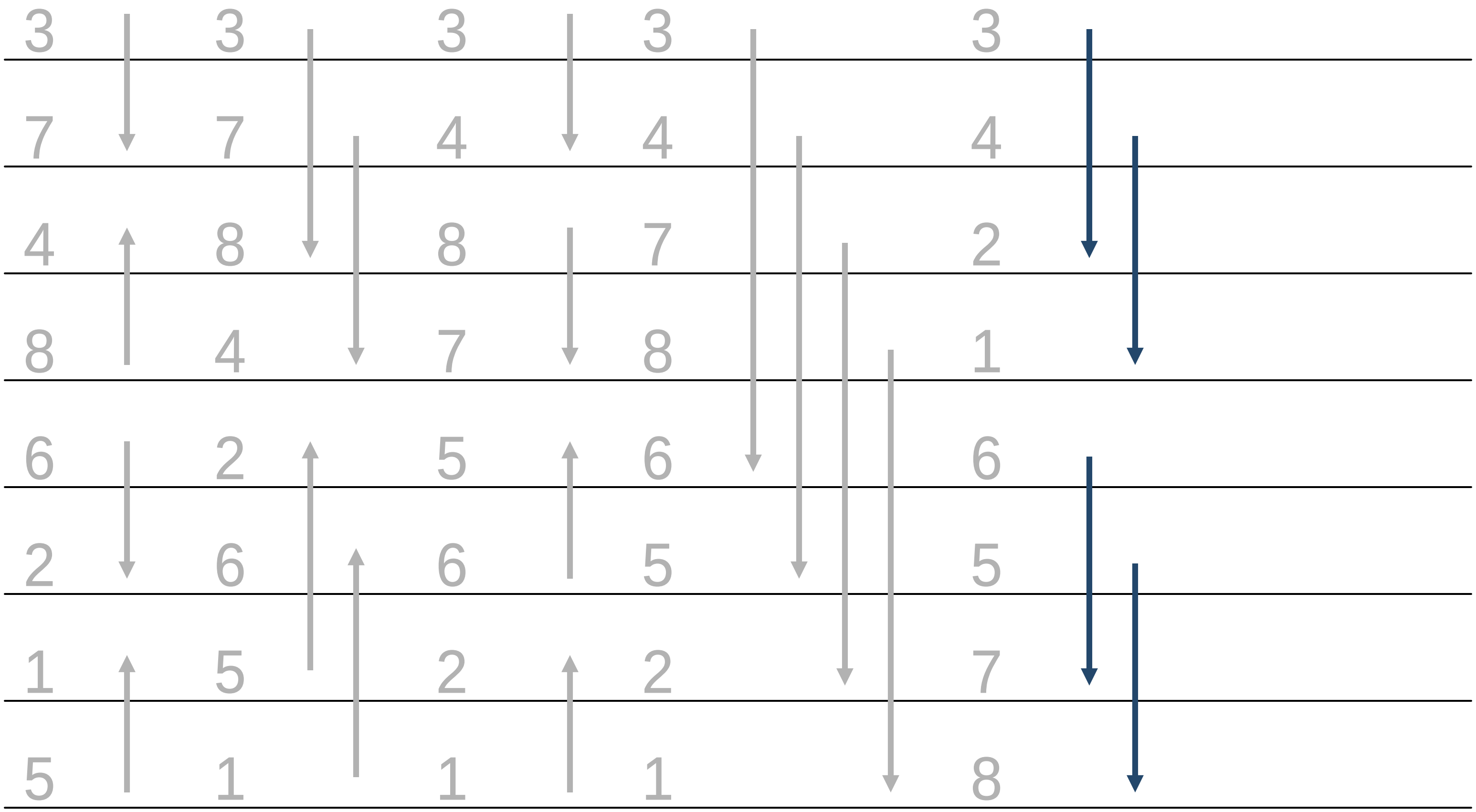
Bitonic Merge Sort



3	3	3	3	3
7	7	4	4	4
4	8	8	7	2
8	4	7	8	1
6	2	5	6	6
2	6	6	5	5
1	5	2	2	7
5	1	1	1	8

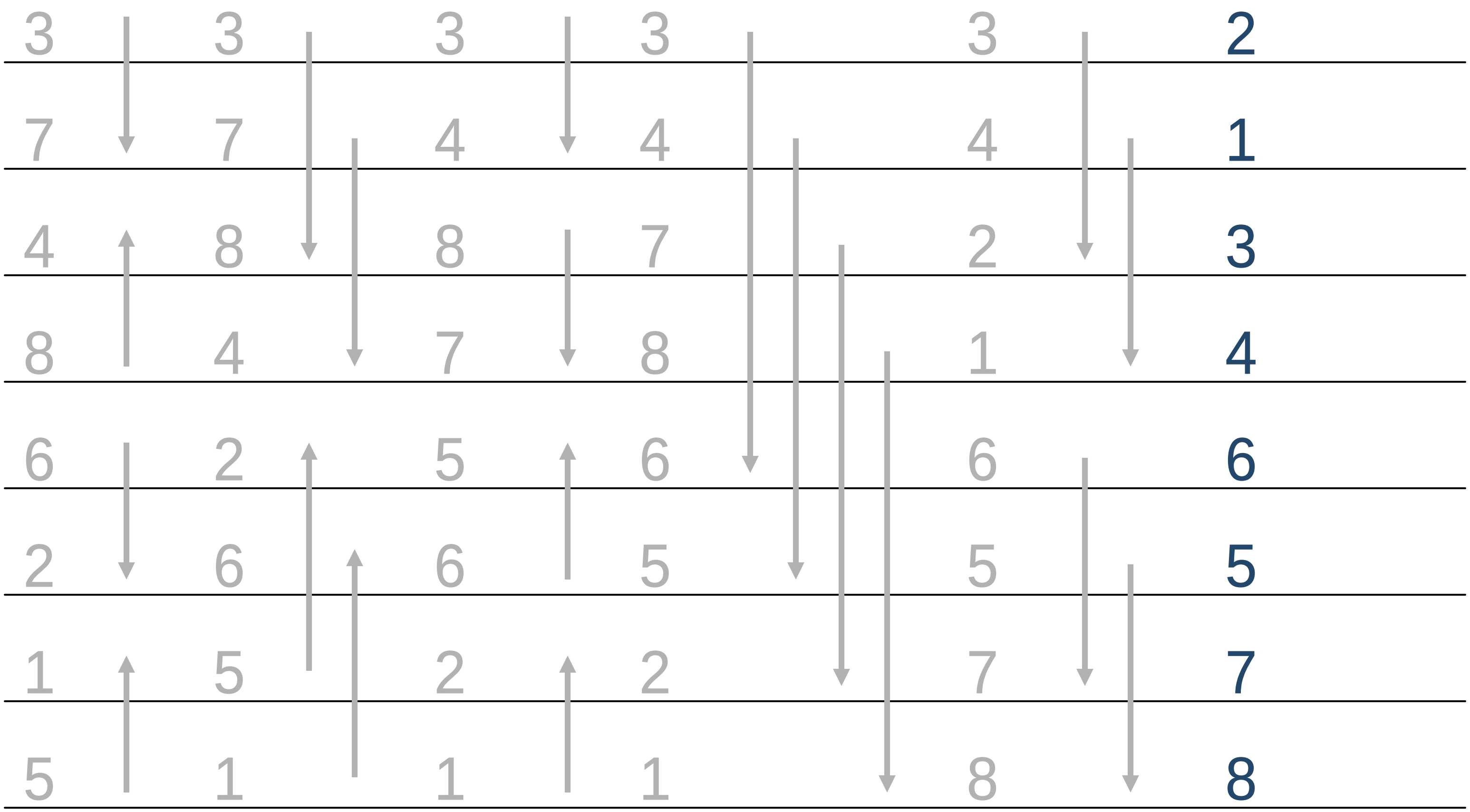
Sort the bitonic list

Bitonic Merge Sort



Sort the bitonic list

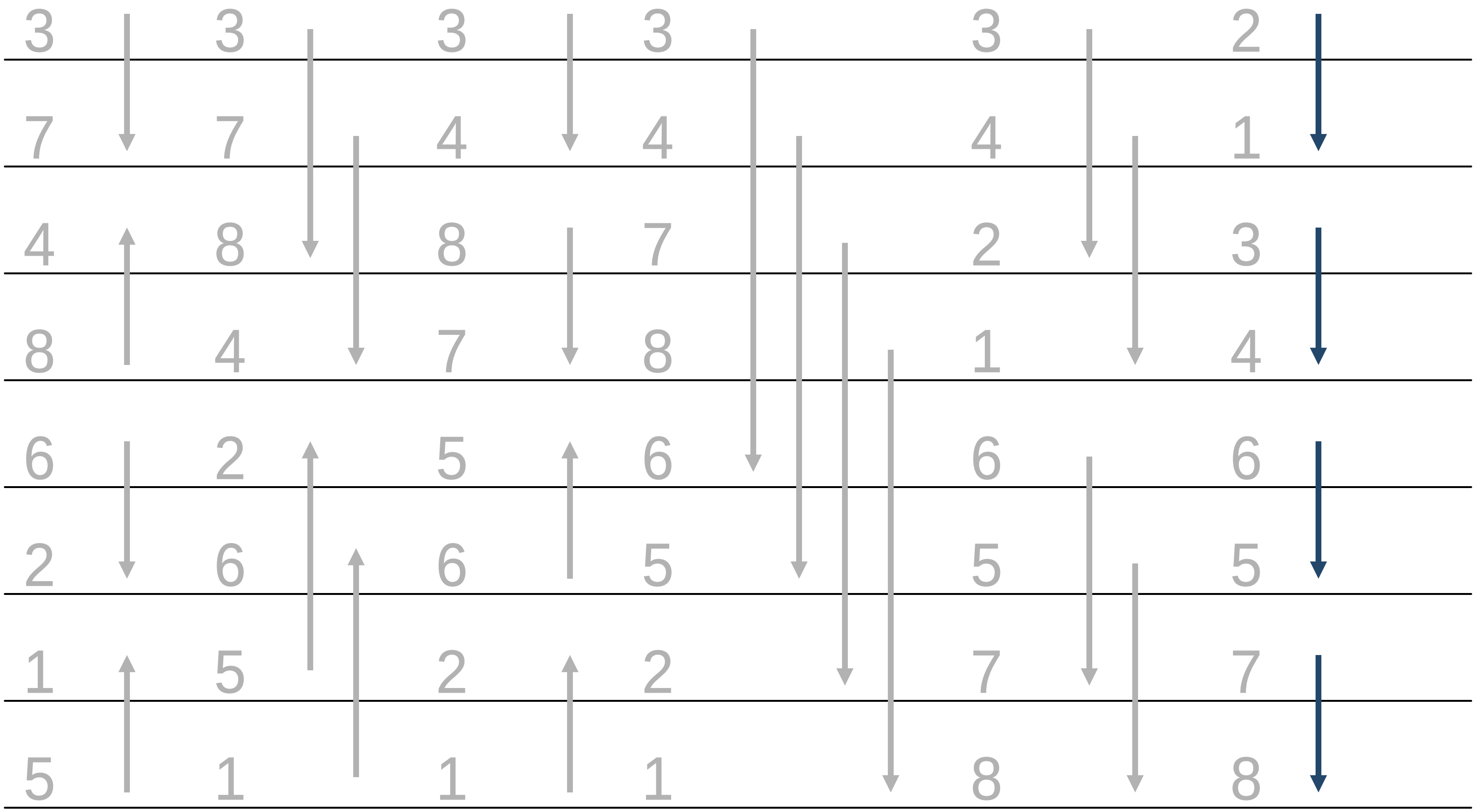
Bitonic Merge Sort



3	3	3	3	3	2
7	7	4	4	4	1
4	8	8	7	2	3
8	4	7	8	1	4
6	2	5	6	6	6
2	6	6	5	5	5
1	5	2	2	7	7
5	1	1	1	8	8

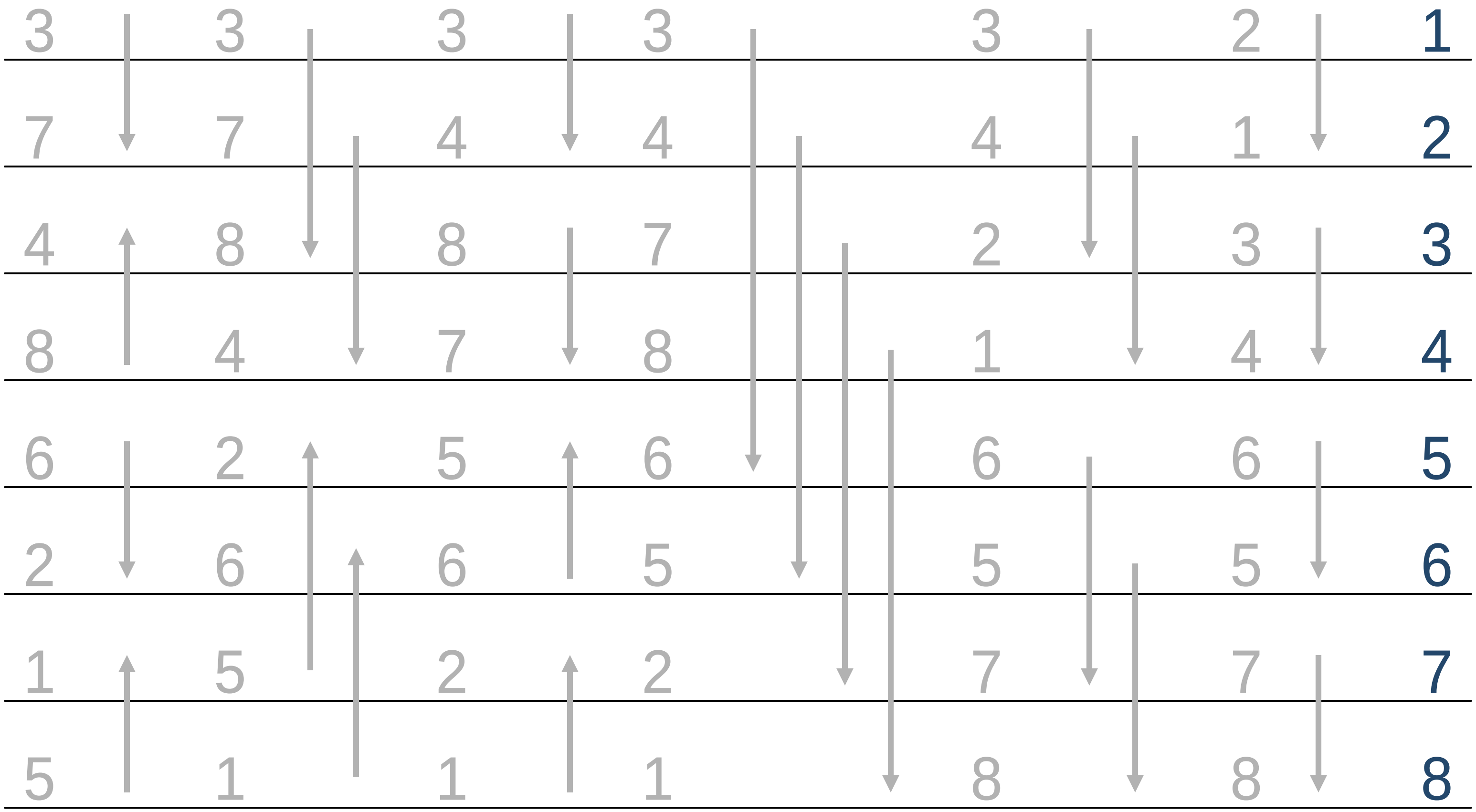
Sort the bitonic list

Bitonic Merge Sort



Sort the bitonic list

Bitonic Merge Sort



3	3	3	3	3	2	1
7	7	4	4	4	1	2
4	8	8	7	2	3	3
8	4	7	8	1	4	4
6	2	5	6	6	6	5
2	6	6	5	5	5	6
1	5	2	2	7	7	7
5	1	1	1	8	8	8

Done!

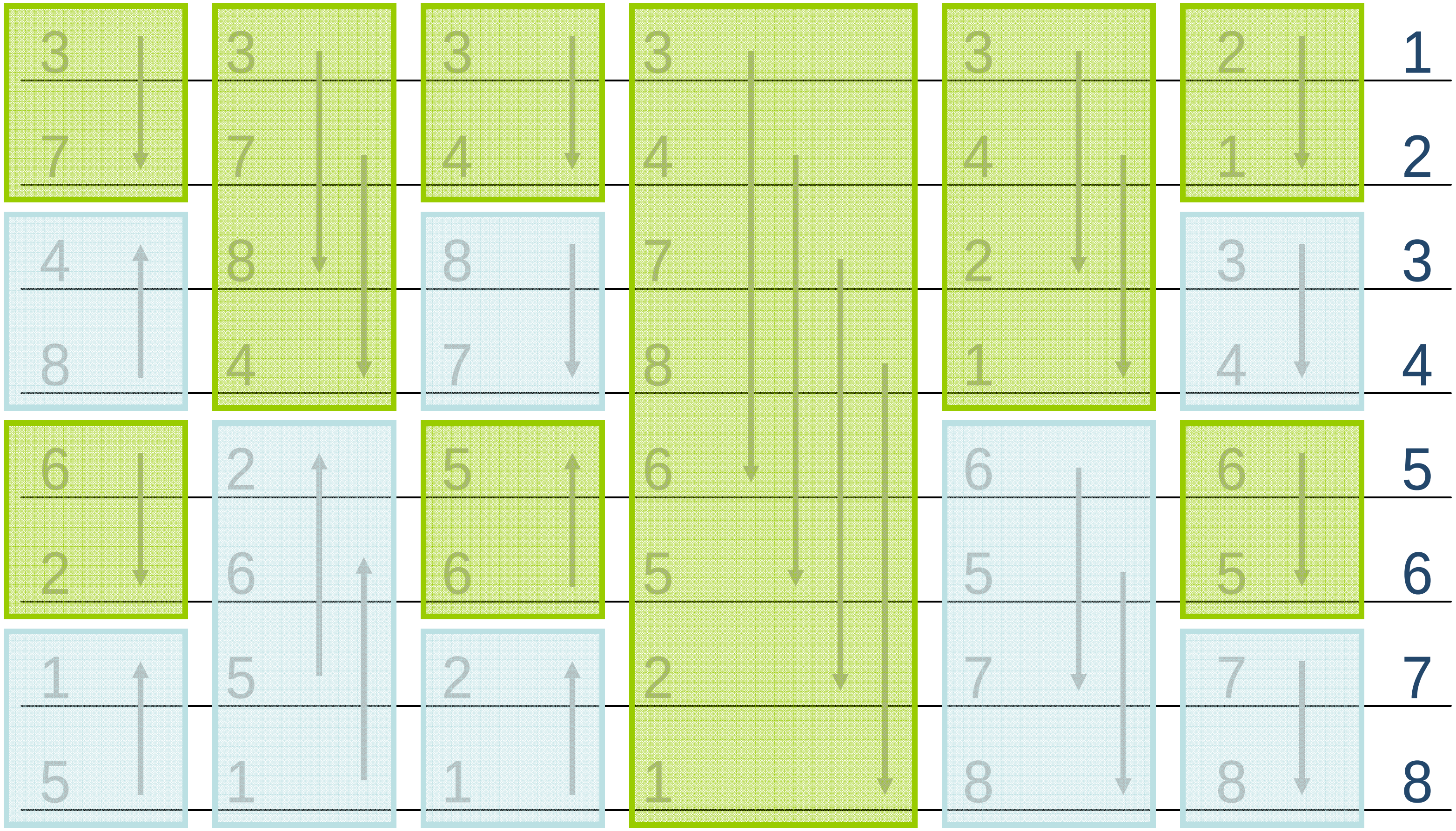
Bitonic Merge Sort Summary

- **Separate rendering pass for each set of swaps**
  - $O(\log^2 n)$ passes
  - Each pass performs n compare/swaps
  - Total compare/swaps: $O(n \log^2 n)$
    - Limitations of GPU cost us factor of logn over best CPU-based sorting algorithms

Making GPU Sorting Faster

- **Draw several quads with similar computation instead of single quad**
  - Reduce decision making in fragment program
- **Push work into vertex processor and interpolator**
  - Reduce computation in fragment program
- **More than one compare/swap per sort kernel invocation**
  - Reduce computational complexity

Grouping Computation



3	3	3	3	3	2	1
7	7	4	4	4	1	2
4	8	8	7	2	3	3
8	4	7	8	1	4	4
6	2	5	6	6	6	5
2	6	6	5	5	5	6
1	5	2	2	7	7	7
5	1	1	1	8	8	8

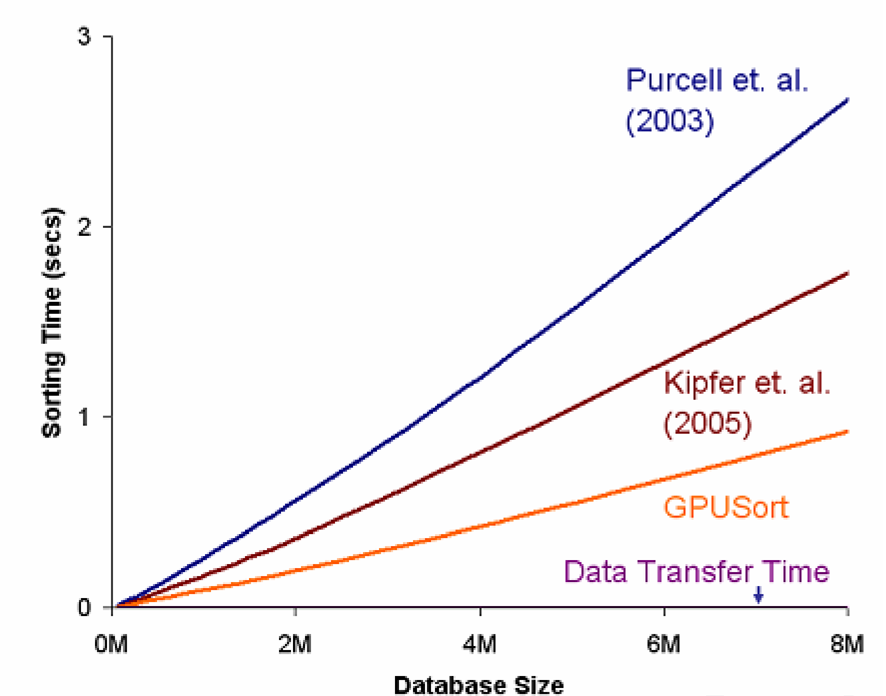
Implementation Details

- **Specify interpolants for smaller quads**
  - ‘down’ or ‘up’ compare and swap
  - distance to comparison partner

- **See Kipfer & Westermann article in GPU Gems 2 and Kipfer et al. Graphics Hardware 04 for more details**

GPU Sort

- Use blending operators for comparison
- Use texture mapping hw to map sorting op.



[Govindaraju 05]

Searching

Types of Search

- **Search for specific element**
  - Binary search
- **Search for nearest element(s)**
  - k-nearest neighbor search

- **Both searches require ordered data**

Binary Search

- **Find a specific element in an ordered list**
- **Implement just like CPU algorithm**
  - Assuming hardware supports long enough shaders
  - Finds the first element of a given value ***v***
    - If ***v*** does not exist, find next smallest element > ***v***
- **Search algorithm is sequential, but many searches can be executed in parallel**
  - Number of pixels drawn determines number of searches executed in parallel
    - 1 pixel == 1 search

Binary Search

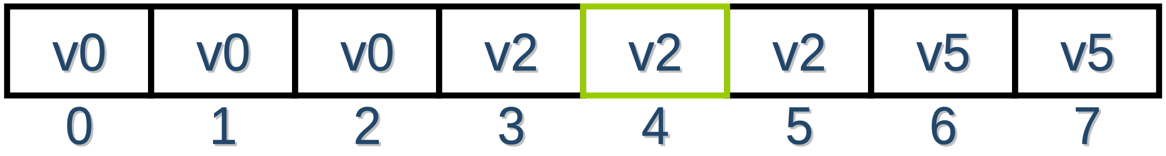
- **Search for v0**

Initialize 4

Search starts at center of sorted array

v2 >= v0 so search left half of sub-array

Sorted List



v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

Binary Search

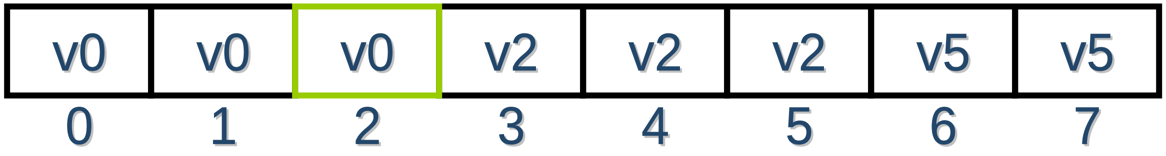
- **Search for v0**

Initialize 4

Step 1 2

v0 >= v0 so search left half of sub-array

Sorted List



v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

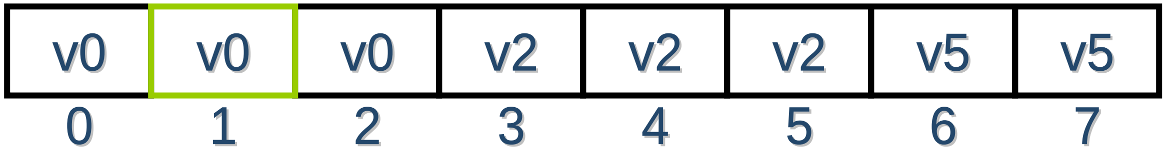
Binary Search

- **Search for v0**

Initialize	4
Step 1	2
Step 2	1

v0 >= v0 so search left half of sub-array

Sorted List



v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

Binary Search

- **Search for v0**

Initialize	4
Step 1	2
Step 2	1
Step 3	0

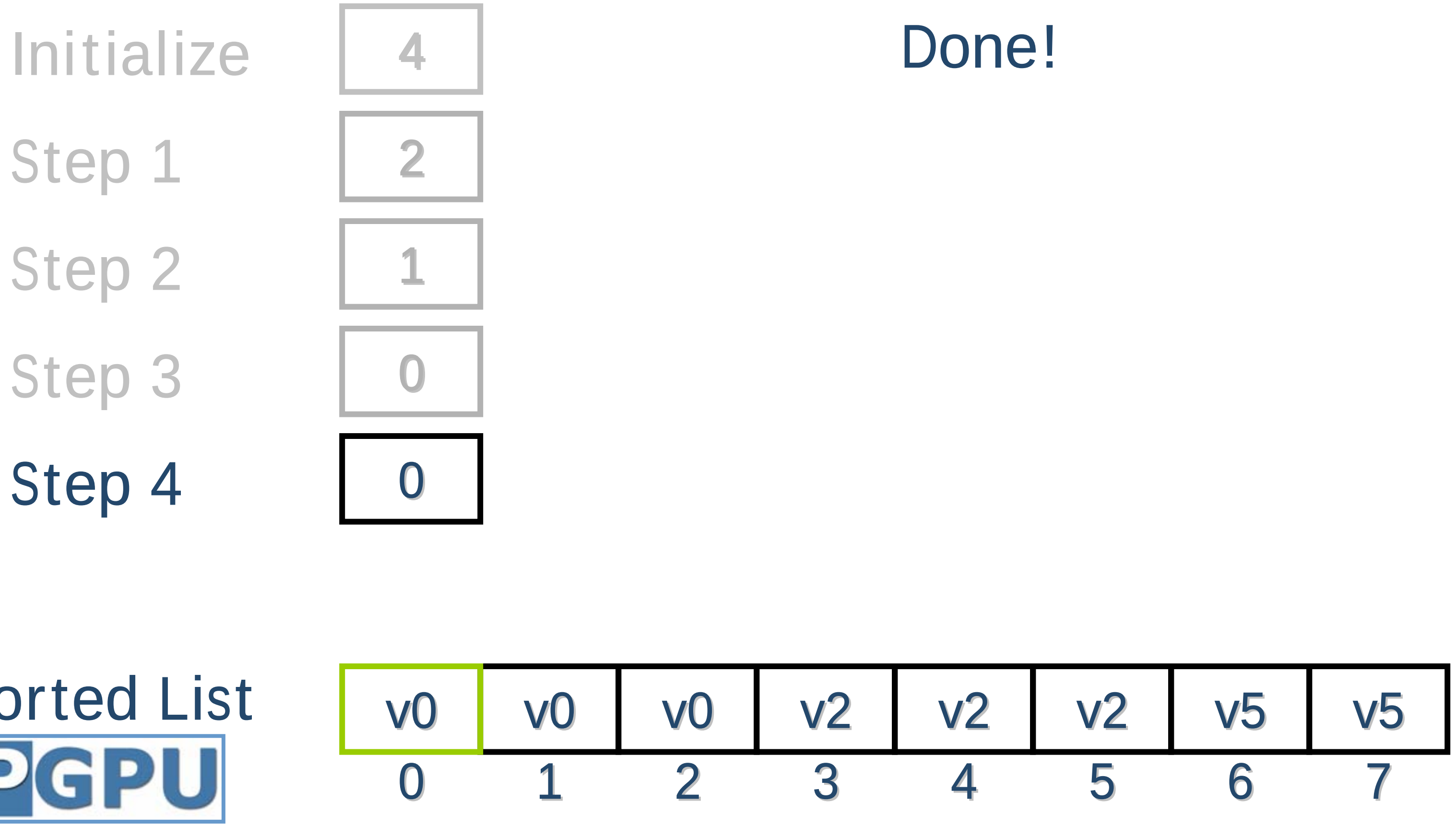
At this point, we either have found v0 or are 1 element too far left

One last step to resolve

Sorted List	v0	v0	v0	v2	v2	v2	v5	v5
	0	1	2	3	4	5	6	7

Binary Search

- **Search for v0**



Initialize	4
Step 1	2
Step 2	1
Step 3	0
Step 4	0

Done!

Sorted List

v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

Binary Search

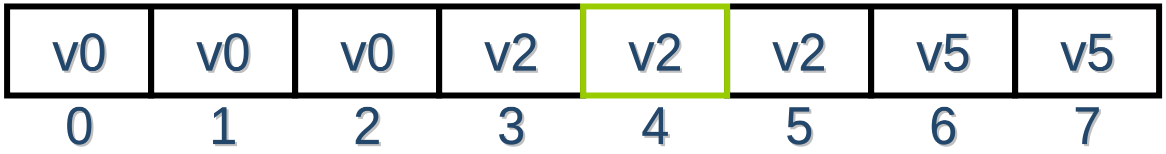
- **Search for v0 and v2**

Initialize 4 4

Search starts at center of sorted array

Both searches proceed to the left half of the array

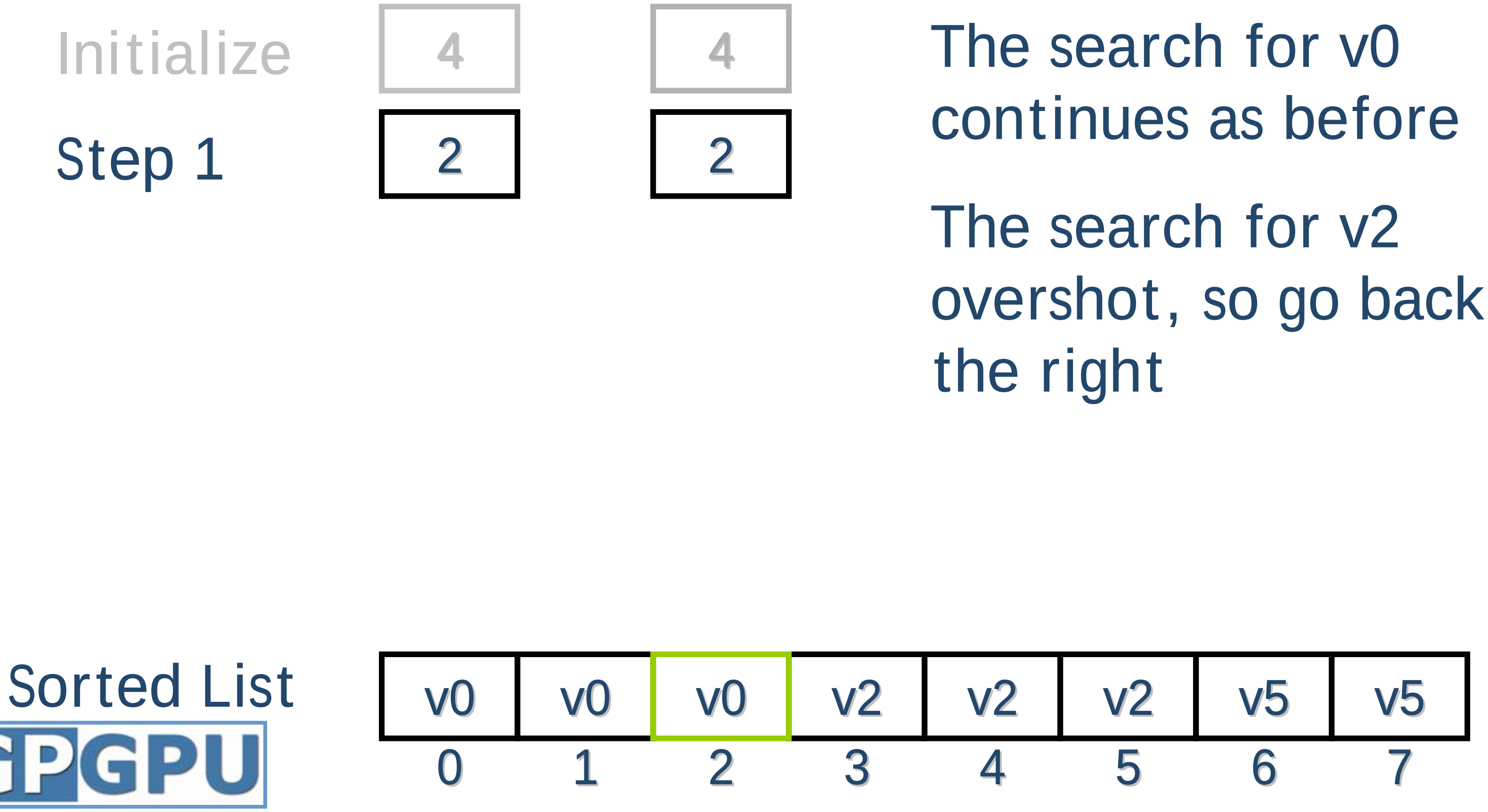
Sorted List



v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

Binary Search

- **Search for v0 and v2**



Initialize	4	4
Step 1	2	2

The search for v0 continues as before

The search for v2 overshot, so go back to the right

Sorted List

v0	v0	v0	v2	v2	v2	v5	v5
0	1	2	3	4	5	6	7

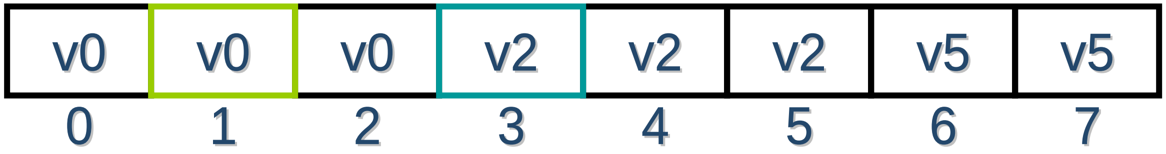
Binary Search

- **Search for v0 and v2**

Initialize	4	4
Step 1	2	2
Step 2	1	3

We’ve found the proper v2, but are still looking for v0

Both searches continue



Sorted List

Sorted List	v0	v0	v0	v2	v2	v2	v5	v5
	0	1	2	3	4	5	6	7

Binary Search

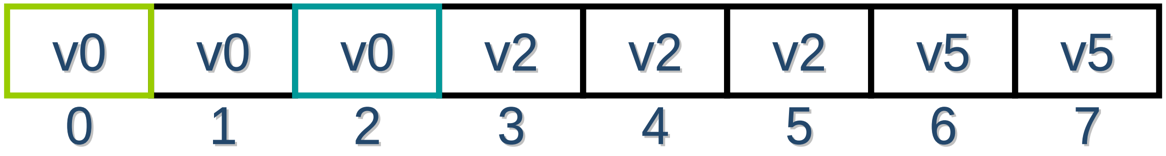
- **Search for v0 and v2**

Initialize	4	4
Step 1	2	2
Step 2	1	3
Step 3	0	2

Now, we’ve found the proper v0, but overshot v2

The cleanup step takes care of this

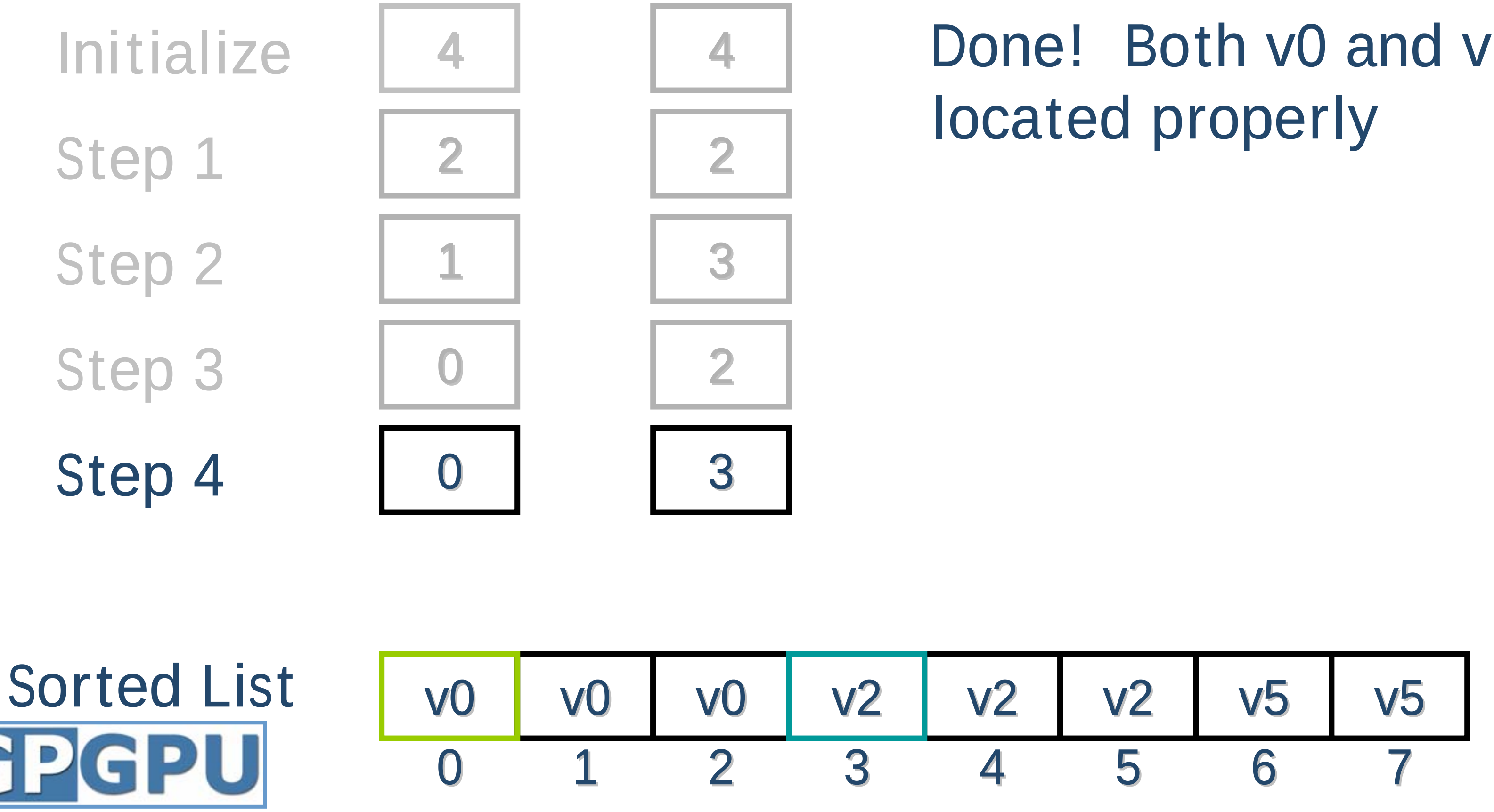
Sorted List



Sorted List	v0	v0	v0	v2	v2	v2	v5	v5
	0	1	2	3	4	5	6	7

Binary Search

- **Search for v0 and v2**



Initialize	4	4
Step 1	2	2
Step 2	1	3
Step 3	0	2
Step 4	0	3

Done! Both v0 and v2 are located properly

Sorted List	v0	v0	v0	v2	v2	v2	v5	v5
	0	1	2	3	4	5	6	7

Binary Search Summary

- **Single rendering pass**
  - Each pixel drawn performs independent search
- **O(log n) steps**

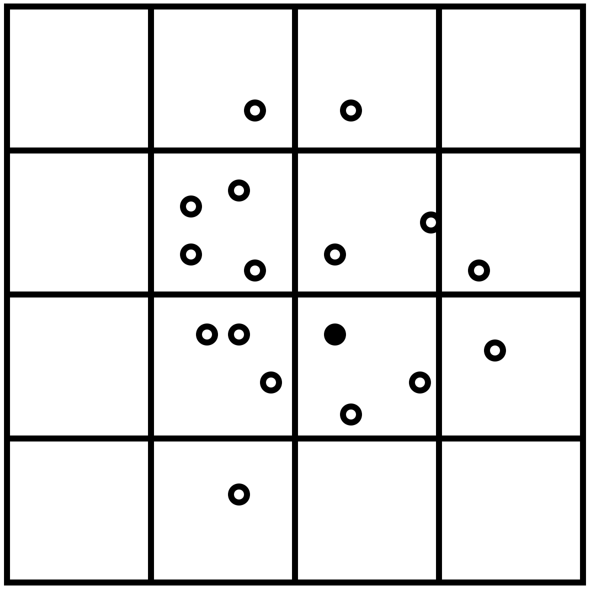
Nearest Neighbor Search

Nearest Neighbor Search

- **Given a sample point *p*, find the *k* points nearest *p* within a data set**

- **On the CPU, this is easily done with a heap or priority queue**
  - Can add or reject neighbors as search progresses
  - Don’t know how to build one efficiently on GPU
- **kNN-grid**
  - Can only add neighbors…

kNN-grid Algorithm



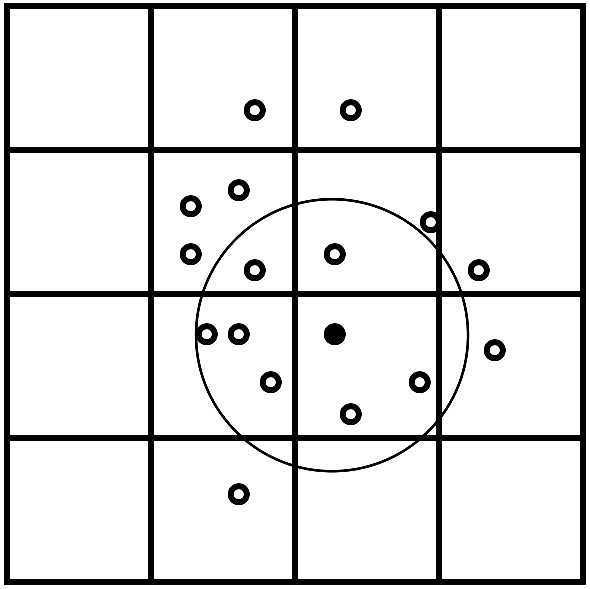
● sample point

● candidate neighbor

● neighbors found

Want 4 neighbors

kNN-grid Algorithm

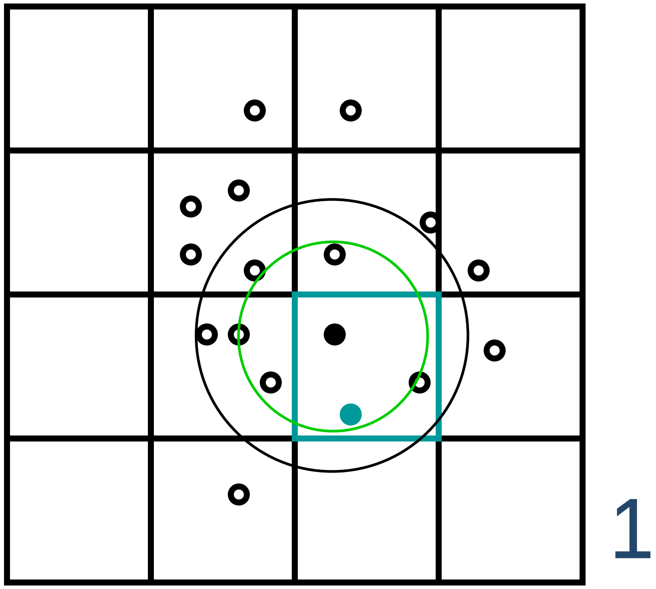


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- Candidate neighbors must be within max search radius
- Visit voxels in order of distance to sample point

kNN-grid Algorithm

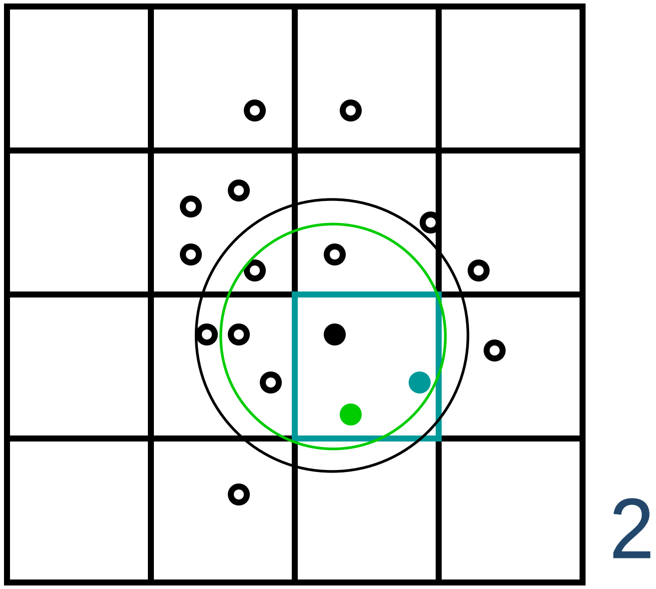


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- If current number of neighbors found is less than the number requested, grow search radius

kNN-grid Algorithm

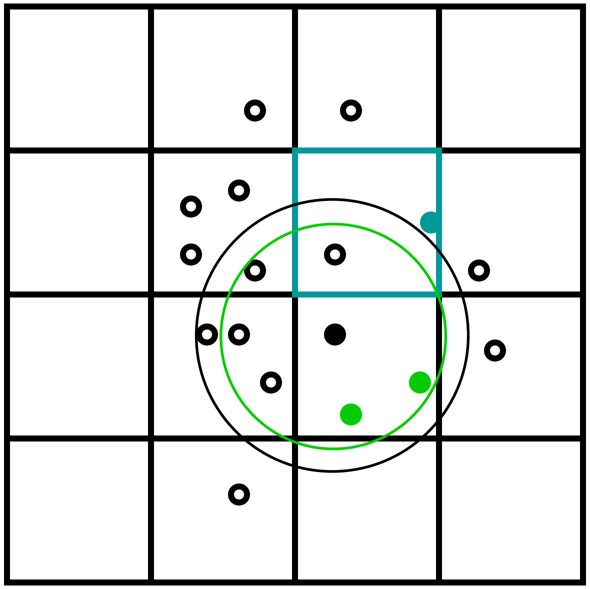


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- **If current number of neighbors found is less than the number requested, grow search radius**

kNN-grid Algorithm

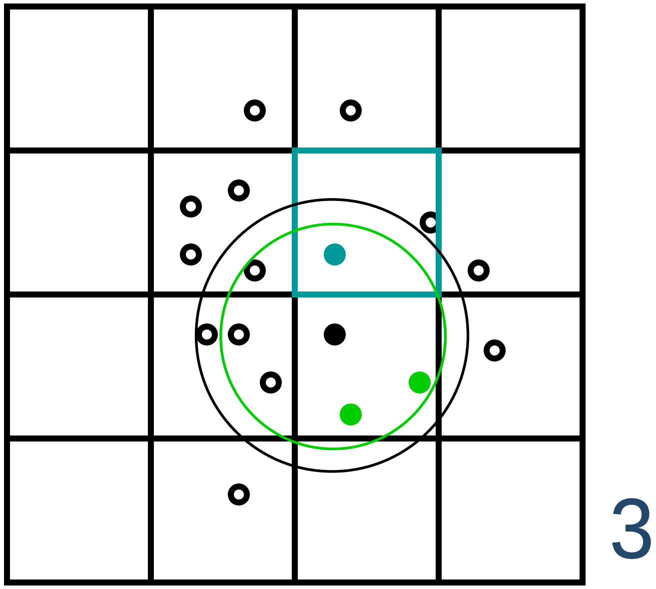


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- Don’t add neighbors outside maximum search radius
- Don’t grow search radius when neighbor is outside maximum radius

kNN-grid Algorithm

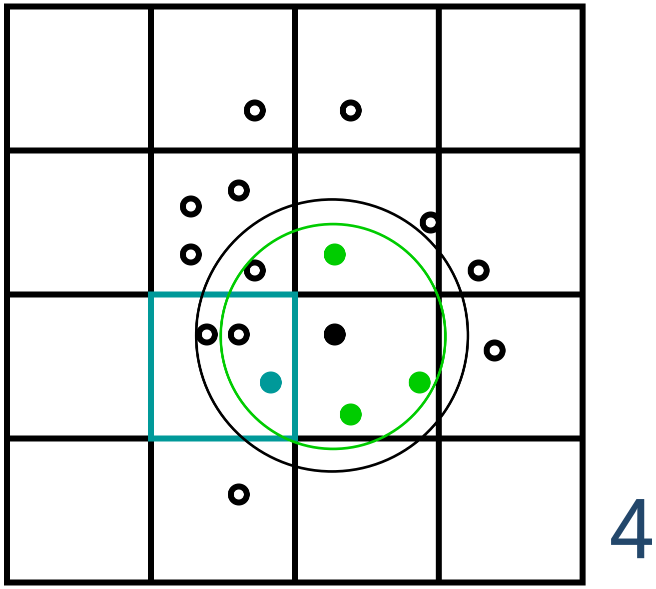


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- Add neighbors within search radius

kNN-grid Algorithm

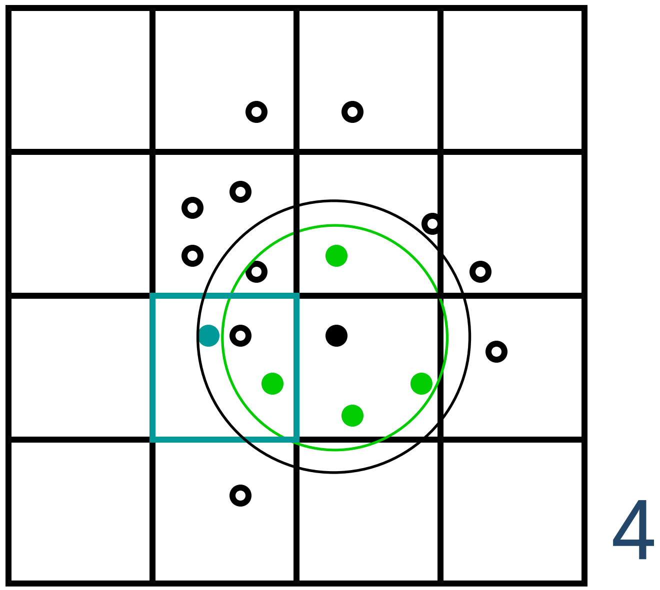


- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

- Add neighbors within search radius

kNN-grid Algorithm

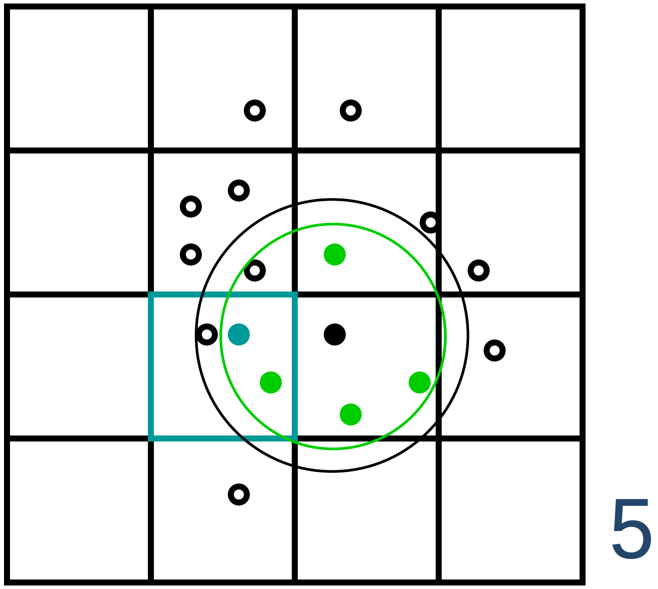


- Don’t expand search radius if enough neighbors already found

- ● sample point
- ● candidate neighbor
- ● neighbors found

Want 4 neighbors

kNN-grid Algorithm

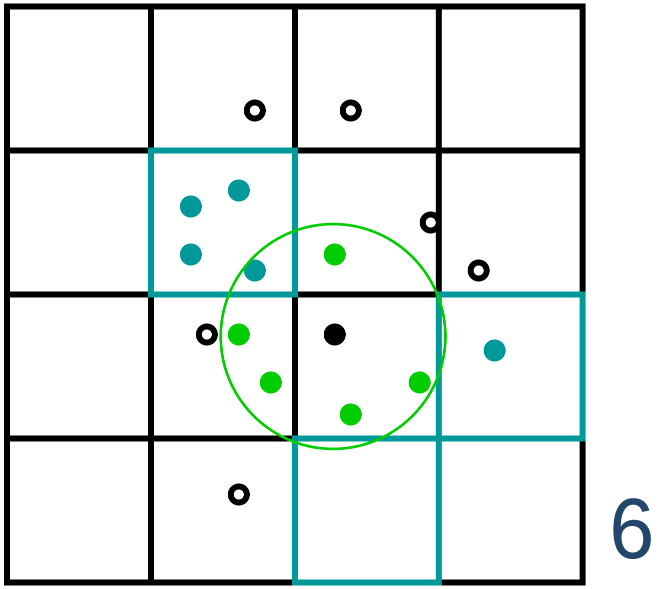


● sample point
● candidate neighbor
● neighbors found

Want 4 neighbors

- Add neighbors within search radius

kNN-grid Algorithm

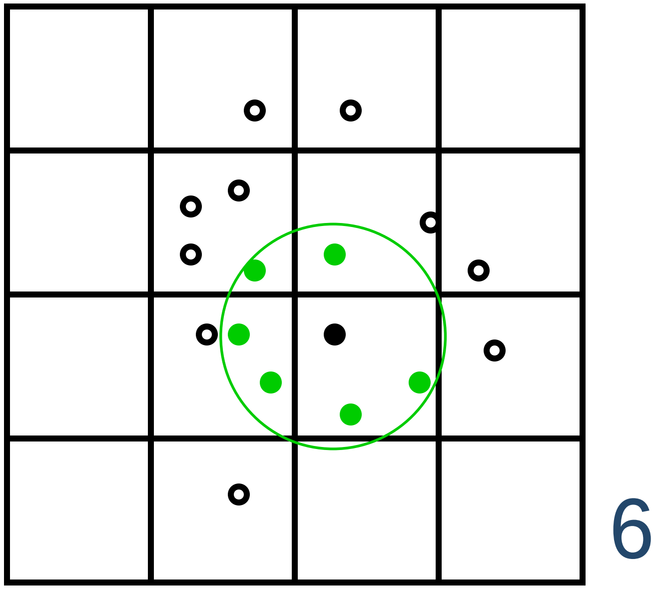


● sample point
● candidate neighbor
● neighbors found

Want 4 neighbors

- Visit all other voxels accessible within determined search radius
- Add neighbors within search radius

kNN-grid Summary



● sample point

● candidate neighbor

● neighbors found

Want 4 neighbors

- Finds all neighbors within a sphere centered about sample point
- May locate more than requested *k*-nearest neighbors

Stream Filtering

- **Select a subset of elements from a list, discard the rest**
  - Irregular reduction operation
- **Stream compaction [Horn 05]**
  - Combine scan and search to compute filtered list
  - O(log n) passes

Stream Filtering

Input Stream

p4	X	p3	p1	X	X	p4	p2

Compacted Stream

p4	p3	p1	p4	p2

X – stream element to remove

Stream Filtering

Input Stream	p4	X	p3	p1	X	X	p4	p2
Scan	0	1	1	1	2	3	3	3
Compacted Stream	p4	p3	p1	p4	p2			

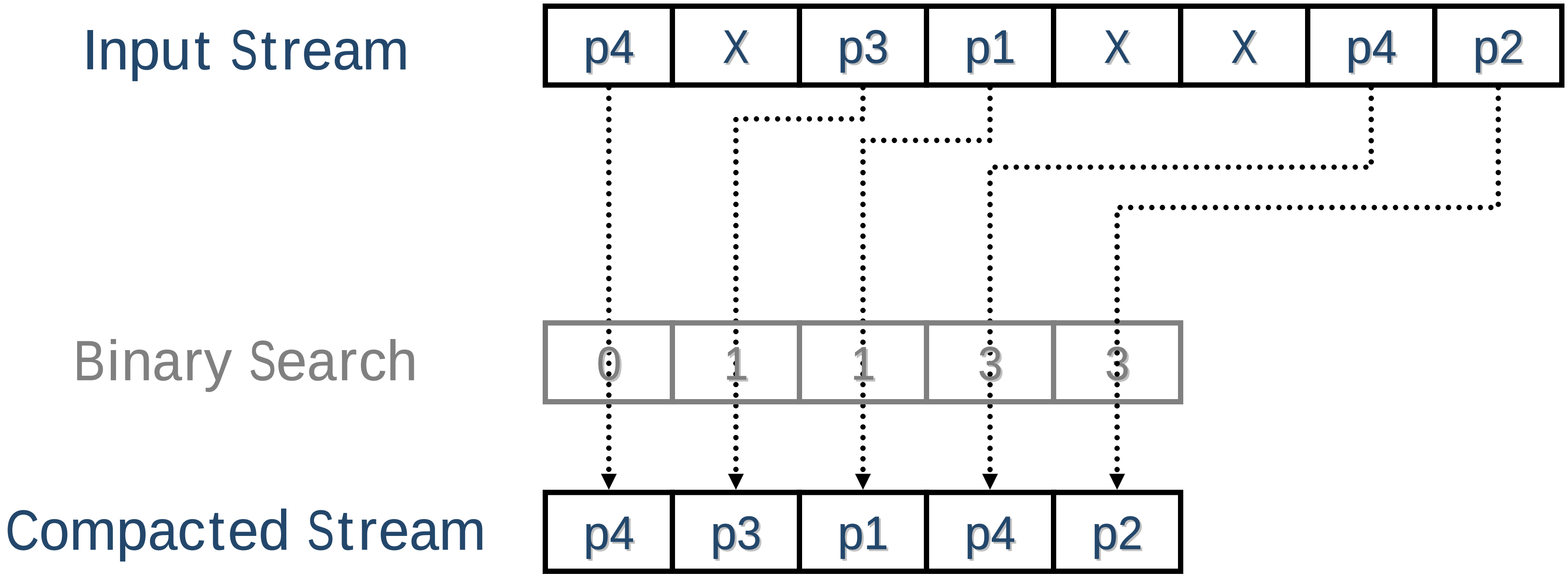
Scan step generates running count of Xs seen so far

Stream Filtering

Input Stream	p4	X	p3	p1	X	X	p4	p2
Scan	0	1	1	1	2	3	3	3
Binary Search	0	1	1	3	3			
Compacted Stream	p4	p3	p1	p4	p2			

Binary search over scan results.
Search result is offset to pull record from

Stream Filtering



Input Stream	p4	X	p3	p1	X	X	p4	p2
Binary Search	0	1	1	3	3			
Compacted Stream	p4	p3	p1	p4	p2			

Stream Filtering Summary

- Combine scan and binary search to compute filtered list
- O(log n) passes